Functional Equivalence between Fuzzy PID and Traditional 2DOF PID Controllers

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Abstract. This paper demonstrates that under certain conditions a class of fuzzy PID controllers are functionally equivalent to a class of traditional two-degree-of-freedom (2DOF) PID controllers. Furthermore, although nonlinearities can be integrated to a traditional 2DOF PID controller, its fuzzy counterpart is intrinsically nonlinear. These nonlinearities, reside in the fuzzy rule base. Although fine tuning can be achieved in both traditional and fuzzy PID controllers, the latest one is superior due to that non-linear control surface that is obtained by modifying the parameters that define the fuzzy rules set. The findings are demonstrated by simulating two benchmark processes taken from the literature.

Keywords. PID control, Fuzzy control, 2DOF PID control, nonlinear control.

1. Introduction

Due to their simple structure, traditional proportional-integral-derivative (PID) controllers continue to be the most adopted controllers in practical cases [1-4]. Furthermore, they are relatively easy to tune and their basic structure is well understood by engineers and industrial practitioners [5-7].

Over time, fuzzy logic control (FLC) has been widely used in industrial processes [8, 9]. These applications exploit the heuristic nature of FLC for both linear and non-linear systems. In particular, due to the success of traditional PID control, several structures of PID-type FLC (PID-FLC) have been proposed and studied (including PI and PD) [10-14]. As a result, several approaches have investigated the relationship between traditional PID control and PID-FLC [12, 14-16].

The degree of freedom of a controller is determined by the number of closed-loop transfer functions that can be adjusted independently [17]. Due that 2DOF PID control offers natural advantages over one-degree-of-freedom PID control, various 2DOF PID controllers have been proposed in the literature [17-19]. Similarly, there have been proposed 2DOF FLC [20, 21]. However there are not related to their traditional 2DOF PID counterpart. In this work there is demonstrated that under certain conditions a class of PID-FLC is functionally equivalent to a class of traditional 2DOF PID control. In addition, the main advantage of the PID-FLC over its traditional counterpart is that a nonlinear control surface can be achieved through the manipulation of the parameters that define the fuzzy rule set.

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The paper is organized as follows. In Section 2 the functional equivalence between 2DOF PID and fuzzy PID controllers is presented. In Section 3 simulation of two benchmark processes taken from the literature is developed in order to demonstrate the findings. Conclusions are drawn in Section 4.

2. Functional equivalence between 2DOF PID and fuzzy PID controllers

2.1 Traditional PID control

The traditional PID controller has the following standard form in the time domain:

$$u(t) = K_p \left[e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right] \tag{1}$$

where: $u(t)$ is the control action, $e(t)$ is the system error, K_p is the proportional gain, T_i is the integral time constant and T_d is the derivative time constant. Also (1) can be written as:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau - K_d \frac{de(t)}{dt} \tag{2}$$

where $K_i = K_p/T_i$ and $K_d = K_pT_d$. In this case the tuning problem consists in selecting the values of these three parameters.

2.2 DOF PID control

Although several equivalent forms of 2DOF PID controllers have been proposed [20, 21], in this work the one proposed by Panagopoulose, et al. [22] is utilized, as is illustrated in Fig. 1. From this figure, the process transfer function G(s) is controlled with a PID controller with two degrees of freedom. The transfer function $G_c(s)$ describes the feedback from process output y to control signal u, and $G_{ff}(s)$ describes the feed forward from set point y_{sp} to u. The external signals that act on the controller loop are the set point y_{sp} and the load disturbance l. Note that for simplicity, measurement noise is not being considered. In this case, the corresponding 2DOF PID controller has the following form in the time domain:

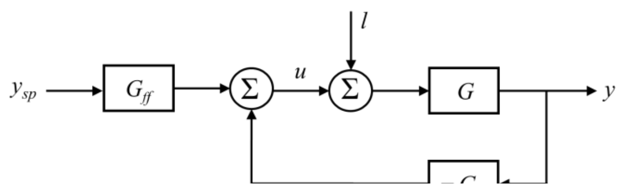

Fig. 1. Block diagram of the 2DOF PID controller.

$$u(t) = K_p\left(by_{sp}(t) - y(t)\right) + K_i\int_0^t\left(y_{sp}(\tau) - y(\tau)\right)d\tau - K_d\left(c\frac{dy_{sp}(t)}{dt} - \frac{dy(t)}{dt}\right) \tag{3}$$

where K_p, K_i , K_d, b and c are the controller tuning parameters.

2.3 Fuzzy PID control

As in traditional control, in fuzzy control there are the analogous structures of the PI type fuzzy logic controller (PI-FLC), PD type fuzzy logic controller (PD-FLC) and the PID type fuzzy logic controller. For the case of the PID-FLC several structures have been proposed. In this work, the one referred to as Modified Hybrid PID-Fuzzy Logic Controller (MHPID-FLC) is adopted. In this structure a combination of a PI-FLC and a PD-FLC is used to implement a PID-FLC with a common two-dimensional rule base, as is shown in Fig. 2(a). Therefore, once appropriate scaling factors G_E, $G_{\Delta E}$, $G_{\Delta U}$ and G_U are selected, a PID control strategy is implemented by combining a PI incremental algorithm and a PD positional algorithm using a two-term fuzzy control rule base.

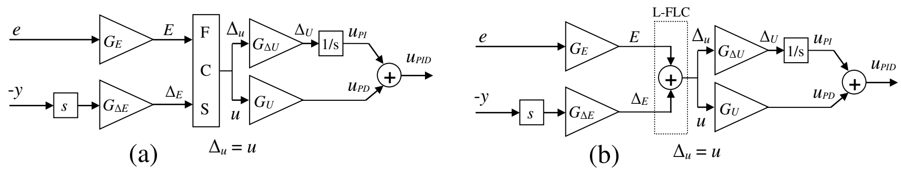

Fig. 2. (a) Schematic representation of the MHPID-FLC; (b) Simplified structure.

2.4 Functional equivalence

This section demonstrate that under certain conditions the 2DOF PID control and the MHPID-FLC are functionally equivalent. Let's define the next set of conditions:

1. The Fuzzy Control System (FCS) inside the MHPID-FLC structure is a first-order Sugeno fuzzy model [23], with fuzzy rules of the form:

   If E is A and Δ_E is B then $u = pE + q\Delta_E + r$

   where A and B are fuzzy sets in the antecedent, while p, q, and r are all constants.
2. The FCS rule base consists of four rules:

   R1: If E is N and Δ_E is N then u = $p_1E + q_1\Delta_E + r_1$
   R2: If E is N and Δ_E is P then u = $p_2E + q_2\Delta_E + r_2$
   R3: If E is P and Δ_E is N then u = $p_3E + q_3\Delta_E + r_3$
   R4: If E is P and Δ_E is P then u = $p_4E + q_4\Delta_E + r_4$

   where the coefficient constants $p_i = q_i = 1$, and $r_i = 0$; for i = 1, 2, 3, 4. The linguistic labels for the fuzzy sets are defined as P = Positive and N = Negative.
3. The universe of discourse for both FCS inputs is normalized on the range [-1, 1].
4. The membership functions of the input variables, E and Δ_E, to the FCS are triangular complementary fuzzy sets [24], and they are defined as shown in Fig. 3(a).

5. The product-sum compositional rule of inference [25] is used in the stage of rule evaluation.
6. The weighted average method is used in the defuzzification process.

If all the above conditions are satisfied, then the 2DOF PID controller and the MHPID-FLC are functionally equivalent. Note that under assumptions 1-6 the FCS inside the MHPID-FLC structure is the simplest that can be considered, and its output is simply given by the sum of its inputs. This FCS is known as the normalized and linear Fuzzy Logic Controller (L-FLC); its control surface is shown in Fig. 3(b). This simplifies the structure of the MHPID-FLC as is shown in Fig. 2(b).

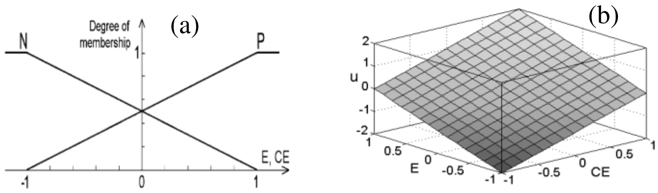

Fig. 3. (a) Membership functions of the L-FLC, (b) Control surface of the L-FLC

Therefore, from Fig. 2(b), the output of the MHPID-FLC is given as (for simplicity the time dependence is not denoted):

$$u_{PID} = u_{PI} + u_{PD} \tag{4}$$

$$u_{PID} = G_{\Delta U} \int \left[G_E e - G_{\Delta E} \frac{dy}{dt} \right] + G_U \left[G_E e - G_{\Delta E} \frac{dy}{dt} \right] \tag{5}$$

From here, performing operations an grouping terms, it is easy to arrive to:

$$u_{PID} = G_U G_E y_{sp} - \left(G_{\Delta U} G_{\Delta E} + G_U G_E \right) y + G_{\Delta U} G_E \int e - G_U G_{\Delta E} \frac{dy}{dt} \tag{6}$$

Therefore, if (3) and (6) are compared, then it is noted that the MHPID-FLC operates like a traditional 2DOF PID controller with the equivalent set-point weights, proportional, integral and derivative gains given by:

$$c = 0 \tag{7}$$

$$K_p b = G_U G_E \tag{8}$$

$$K_p = G_{\Delta U} G_{\Delta E} + G_U G_E \tag{9}$$

$$K_i = \frac{K_p}{T_i} = G_{\Delta U} G_E \tag{10}$$

$$K_d = K_p T_d = G_U G_{\Delta E} \tag{11}$$

Note that the weighting factor c is considered as zero, as it does not appear in the MHPID-FLC structure. However, the weighted derivative term $K_d c dy_{sp}(t)/dt$ can be added to the MHPID-FLC structure as a separated term, as is shown in Fig. 4. In this figure the term K_d has been replaced by $G_U G_{\Delta E}$ as given by (11). Therefore, by doing that, the controller shown in Fig. 4 is functionally equivalent to the 2DOF PID controller defined by (3) if the defined conditions are fulfilled.

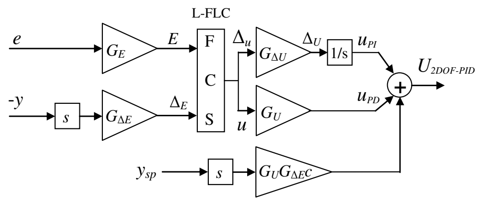

Fig. 4. MHPID-FLC structure with added weighted set-point derivative term.

2.5 Tuning procedure

Given the structure and the functional equivalence described in the previous section, now the problem is how to perform the tuning of the scaling factors G_E, $G_{\Delta E}$, $G_{\Delta U}$, G_U and the weighting factors b and c. If the values of K_p, K_i, and K_d or alternatively the values of K_p, T_i, and T_d are available, then the values G_E, $G_{\Delta E}$, $G_{\Delta U}$ and G_U in the MHPID-FLC structure (see figure 2) can be calculated as follows. First, let's define:

$$G_E = 1 \tag{12}$$

From (12) in (10):

$$G_{\Delta U} = K_i \tag{13}$$

Performing substitutions in (9) and (11) and making operations, it is easy to arrive to the next second order equation:

$$0 = K_p^2 b^2 - K_p^2 b + K_i K_d \tag{14}$$

The solution of this equation is given by:

$$b = \frac{1}{2} \pm \frac{\sqrt{(-K_p^2)^2 - 4K_p^2 K_i K_d}}{2K_p^2} \tag{15}$$

If the Ziegler-Nichols (Z-N) frequency response method is used to find the traditional PID gain parameters, they are given by the set of equations:

$$K_P = 0.6K_u\text{; } K_i = \frac{2K_P}{T_u}\text{; } K_d = \frac{K_p T_u}{8} \tag{16}$$

From this set of equations it is straightforward to demonstrate that:

$$4K_iK_d = K_p^2 \tag{17}$$

Substituting (17) in (15) results in:

$$b = \frac{1}{2} \tag{18}$$

From here the remaining scaling factors are obtained as:

$$G_U = \frac{1}{2}K_p \tag{19}$$

$$G_{\Delta E} = \frac{2K_d}{K_p} \tag{20}$$

It is surprising to find that the value of the set point weighting factor b is uniquely determined as 0.5 when the Z-N tuning method is used and it is intrinsically included in the MHPID-FLC structure, although it is not explicitly included. This simplifies the calculation of the remaining scaling factors. With regard to the weighting factor c of the set point derivative term, it is left as an additional free adjusting factor, which can be manipulated for fine tuning, if needed. Note that the Z-N tuning parameters, K_u and T_u, can be obtained with the relay auto-tuning method [26].

3 Simulation results

In this section the results from the simulation of two bench mark processes taken from the literature are presented. The simulations for each process have been developed in the Matlab/Simulink simulation environment, together with the Fuzzy Logic Toolbox. The Z-N tuning parameters where obtained with the relay auto-tuning method, from there the scaling factors where obtained, as explained in the previous section.

Transfer function of a stable process [27]:

$$G_1(s) = \frac{e^{-0.4s}}{(s+1)^2} \tag{21}$$

Results: The comparison of the set point and load disturbance rejection responses are shown in Fig. 5 (a). Note that the response of the traditional 2DOF PID controller with b=0.5 and c=0, is exactly the same as the obtained with the MHPID-FLC, which demonstrate that they are functionally equivalent. In both cases, further fine tuning can be achieved by adjusting the weighting factor c. However, in the case of the MHPID-FLC additional tuning can be performed by adjusting the parameters that define the fuzzy rules or by modifying the scaling factors, or by modifying all these parameters altogether. As an example of further fine tuning, Fig. 5(a) also shows the comparison of the results obtained when the fuzzy rules have been modified by set-ting the coefficients $p_1=p_4=2.5$, $q_1=q_4=3$, $r_1=r_4=0$, $p_2=p_3=0.4$, $q_2=q_3=0.4$, $r_2=r_3=0$, this controller is referred to as MHPID-FLC-MRS. In the same figure, the results obtained when modifying the scaling factors as $G_E=1$, $G_{\Delta E}=0.735$, $G_U=1.8$ and $G_{\Delta U}=1.8$, also are shown, controller referred to as MHPID-FLC-MSF. In addition, the results of performing additional tuning by adjusting the weighting factor c, for the three controllers, are shown in Fig. 5(b), compared with the original 2DOF PID controller. For better comparison, the obtained integral of the absolute error (IAE) for all the simulated cases are shown in Table 1, the integral is reset after the step response settling time to measure the IAE for the load rejection responses.

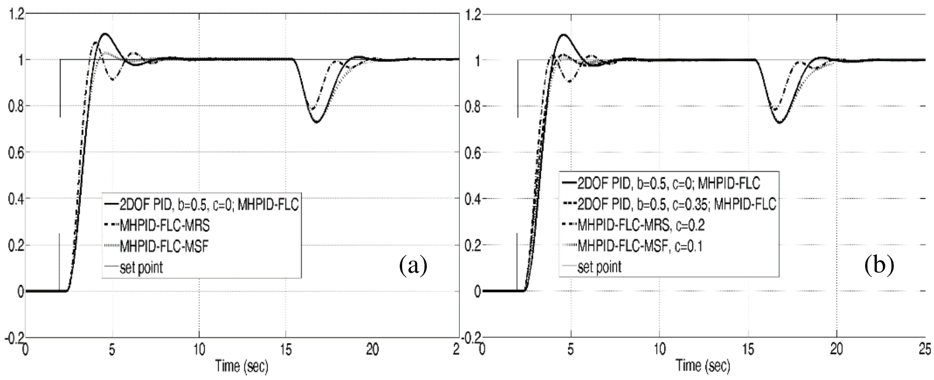

Fig. 5. (a) Step response and load rejection plots for process 1; (b) further tuning through the weighting factor c for process 1.

Table 1. IAE measurements for process $G_1(s)$

	IAE	
	Set point	Load rejection
2DOF PID, b=0.5, c=0; MHPID-FLC	1.4173	0.4739
MHPID-FLC-MRS	1.2533	0.3157
MHPID-FLC-MSF	1.3165	0.5554
2DOF PID, b=0.5, c=0.35; MHPID-FLC	1.2146	0.4739
MHPID-FLC-MRS, c=0.2	1.1816	0.3157
MHPID-FLC-MSF, c=0.1	1.2826	0.5554

Transfer function of an unstable process [27]:

$$G_1(s)=\frac{e^{-0.5s}}{s(s+1)} \tag{22}$$

The comparison of the set point and load disturbance rejection responses are shown in Fig. 6(a). Similarly than for the previous process, the response of the traditional 2DOF PID controller with b=0.5 and c=0, is exactly the same as the obtained with the MHPID-FLC, proving that they are functionally equivalent. Fig. 6(a) also shows the comparison of the results obtained when the fuzzy rules have been modified by set-ting the coefficients p_1=p_4=1, q_1=q_4=1, r_1=r_4=0, p_2=p_3=3.4, q_2=q_3=3.8, r_2=r_3=0, this controller is referred to as MHPID-FLC-MRS. In the same figure, the results obtained when modifying the scaling factors as G_E=1, $G_{\Delta E}$=1.25, G_U=1.5 and $G_{\Delta U}$=0.9, controller referred to as MHPID-FLC-MSF, also are shown. The results of performing additional tuning by adjusting the weighting factor *c*, for the three controllers, are shown in Fig. 6(b), compared with the original 2DOF PID controller. The measured IAE for all the simulated controllers are shown in Table 2.

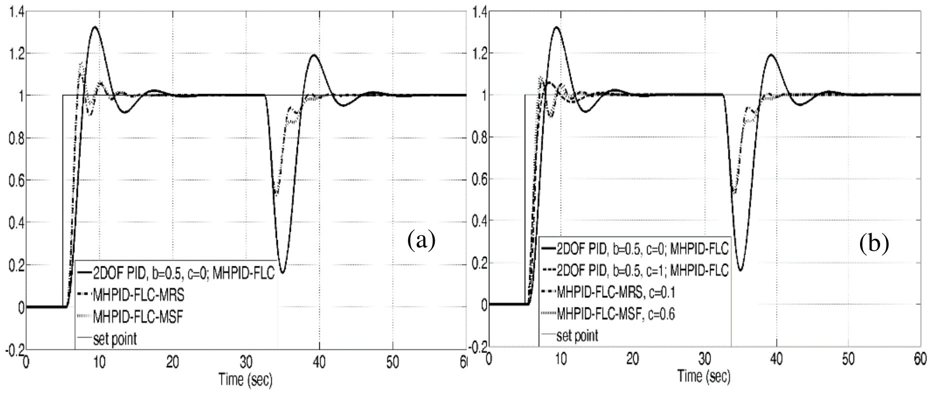

Fig. 6. (a) Step response and load rejection plots for process 2; (b) further tuning through the weighting factor *c* for process 2.

Table 2. IAE measurements for process $G_2(s)$

	IAE	
	Set point	Load rejection
2DOF PID, *b*=0.5, *c*=0; MHPID-FLC	2.9431	3.0544
MHPID-FLC-MRS	1.6324	1.0289
MHPID-FLC-MSF	1.6105	1.1128
2DOF PID, *b*=0.5, *c*=0.35; MHPID-FLC	1.4678	3.0544
MHPID-FLC-MRS, *c*=0.2	1.5853	1.0289
MHPID-FLC-MSF, *c*=0.1	1.3612	1.1128

4 Conclusions

In this work a functional equivalence between 2DOF PID control and MHPID-FLC has been demonstrated. From the simulations performed, the next tuning sequence is recommended for the MHPID-FLC:

1. Find the traditional proportional, integral and derivative gains using the autotuning relay experiment and Z-N formulae.
2. From the traditional proportional, integral and derivative gains calculate the scaling factors G_E, $G_{\Delta E}$, G_U and $G_{\Delta U}$, recall that intrinsically the weighting factor *b*=0.5.

3. Perform fine tuning by adjusting the parameters of the fuzzy rules p_i, q_i, and r_i; for $i = 1, 2, 3, 4$. Or by manipulating the scaling factors G_E, $G_{\Delta E}$, G_U and $G_{\Delta U}$.
4. If needed, further fine tuning can be achieved by manipulating the scaling factor c.

Note, that by modifying the parameters of the fuzzy rules, the control surface of the FCS inside the MHPID-FLC becomes nonlinear. As an example, Fig. 7 shows the control surface obtained when $p_1=p_4=1$, $q_1=q_4=1$, $r_1=r_4=0$, $p_2=p_3=3.4$, $q_2=q_3=3.8$, $r_2=r_3=0$, used for tuning the MHPID-FLC of plant $G_2(s)$. From the tuning procedure it was observed that the only way of improving the load rejection performance was precisely by introducing the nonlinear control surface in the MHPID-FLC. Further adjustment of the weighting factor c only produced a reduction in the step response overshot, but did not produced any change in the load rejection response. Although good results were obtained, more study of the proposed MHPID-FLC and the tuning procedure is needed to fully characterize it. This work is in progress.

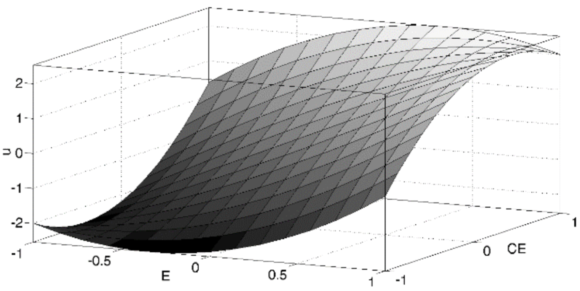

Fig. 7. Nonlinear control surface.

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